**Understanding Calculus:**

**A Junior-High Level Simplification of the Ideas of Derivatives and Integrals**

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Table of Contents

[**Introduction** 3](#_Toc529219259)

[**0.1 Why is understanding the ideas of derivatives and integrals is important for kids at this age?** 3](#_Toc529219260)

[**0.2 What is infinity?** 3](#_Toc529219261)

[**Figure 0.1:** Table Showing Infinite Different Numbers between the Numbers 1 and 2 3](#_Toc529219262)

[**Figure 0.2:** Set of Pictures Showing Circles Being Divided Into Smaller Slices 4](#_Toc529219263)

[**What is a “derivative?”** 5](#_Toc529219264)

[**1.0 A Simplified Definition of a Derivative** 5](#_Toc529219265)

[**1.1 Understanding What a Derivative Is with Visuals** 5](#_Toc529219266)

[**Figure 1.1:** Linear Graph with Slope of -1 5](#_Toc529219267)

[**Figure 1.2:** Graph with Point at (1, 0.1353) 5](#_Toc529219268)

[**Figure 1.3:** A Zoomed-In View of Figure 1.2 6](#_Toc529219269)

[**Figure 1.4:** Pictures Showing More Zoomed-In Views of Figure 1.2 (zoom increases from a to c) 6](#_Toc529219270)

[**Figure 1.5:** Derivatives at Different Parts of Curve 7](#_Toc529219271)

[**Figure 1.6:** Different Slopes for Neighboring Pairs of Infinitely Close Points 7](#_Toc529219272)

[**What is an “integral?”** 8](#_Toc529219273)

[**2.0 A Simplified Definition of an Integral** 8](#_Toc529219274)

[**2.1 Understanding What an Integral Is with Visuals** 8](#_Toc529219275)

# **Introduction**

Within each of us there is a desire to seek knowledge and understand the world around us. Kids especially are constantly pondering about the mysteries of the world around them. Mathematics is one such way of unraveling and understanding those mysteries. You do not think about it, but you, and everyone else, utilize the principles of mathematics every day, whether it be from simply counting the number objects in your hand or designing a full-fledged rocket. Often times, math becomes misconceived with calculations, but math itself is not about calculations. It is about understanding and applying concepts and processes.

## **0.1 Why is understanding the ideas of derivatives and integrals is important for kids at this age?**

Calculus is a branch of mathematics that is revered for its difficulty and rigor; however, its fundamental concepts can be understood by almost anyone, even children. It is especially important that children begin to think about the ideas of calculus because without an understanding of these ideas, understanding its applications will be increasingly more difficult, if not impossible. The rigor associated from the algebra and computations involved in calculus can be learned through time and practice.

## **0.2 What is infinity?**

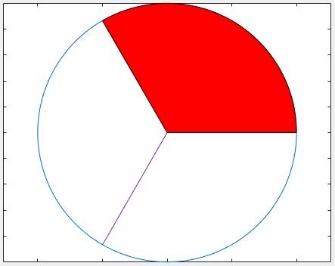
Before we delve into what the concept of derivatives and integrals are, it is important that we take a step back and re-examine the meaning of the word “infinity.” Infinity is sometimes termed as the biggest “number” possible; however, a number is finite. In other words, it has a value and it is limited by that value. For example, the number, 1, has a value and only that value. The number, 2, also has a value and only that value. It is essentially two 1s together. The number, 3, is three 1s together, and so forth. But what is a number with an uncountable large amount of 1s together? That number would be termed as being a number with infinite 1s, but it is not infinity. That number is infinitely large. If finite means to be limited and bounded, then infinite would mean to be unlimited and unbounded; therefore, infinity cannot be a number because it is unbounded and unlimited.

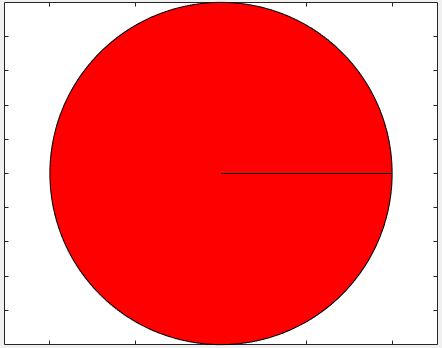
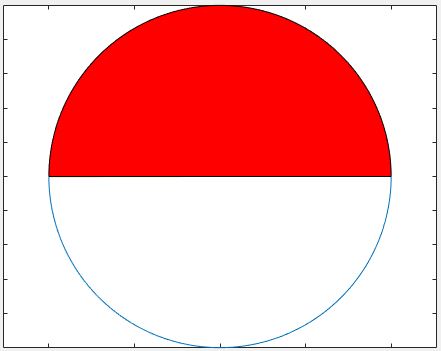
If I asked you to count the total amount of different numbers between one and two, it would be impossible because there exists an infinite amount of numbers in between the numbers, one and two. 1 is a number, but so is 1.1 and 1.11 and 1.111 and so forth. If we keep adding a one to the end of the previous number, we get a different number that is a little bigger than the previous number but still less than 2. As seen in Figure 1 below, if we keep counting the total amount of these numbers before reaching 2, the count will never end. That is the concept of infinity.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Count** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | … |
| **Numbers** | 1 | 1.1 | 1.11 | 1.111 | 1.1111 | 1.11111 | 1.11111 | 1.1111111 | 1.11111111… |

### **Figure 0.1:** Table Showing Infinite Different Numbers between the Numbers 1 and 2

Infinity can also be used to describe something that is extremely small. Let’s take cutting a circle for example. As seen in Figure 2a below, we start with one whole circle, but if we cut it down the middle, we now have half of a circle, or ½. If we divide the circle into thirds, we have three pieces. If we continue cutting the circle, we will get more and more slices, but each slice gets smaller and smaller as seen in the figure below.

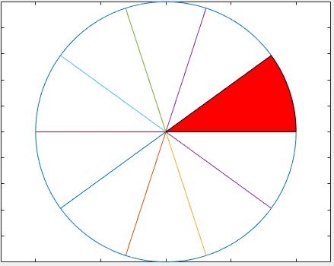
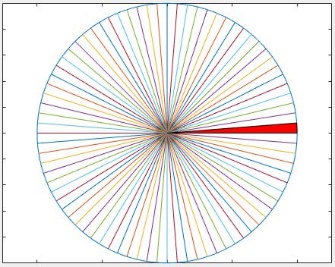
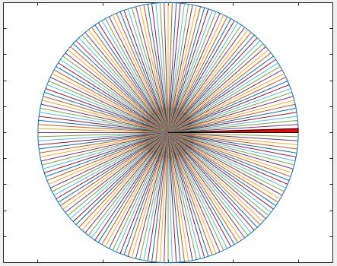




c = 1/3

a = 1

b = 1/2

****

f = 1/200

e = 1/80

d = 1/10

### **Figure 0.2:** Set of Pictures Showing Circles Being Divided Into Smaller Slices

As the slice get smaller, it will eventually reach a point when it is so small that you can’t see it. At that point, it is as if the slice is not there, but we know that it still there. It essentially zero, but not yet at zero. The slice would be 1/∞ of the circle. The slice would then be as described as being infinitely small, or infinitesimal.

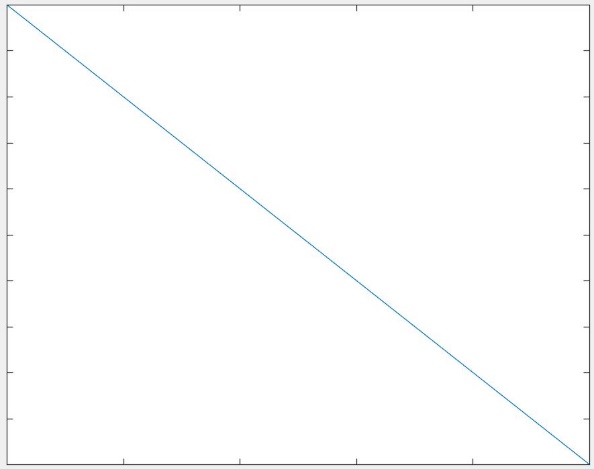
# **What is a “derivative?”**

## **1.0 A Simplified Definition of a Derivative**

A derivative is one of the critical ideas belonging to Calculus, a branch of mathematics. A derivative can be thought of as the change between two infinitely close points. In mathematics, it is thought of as the slope between two infinitely close points on a graph.

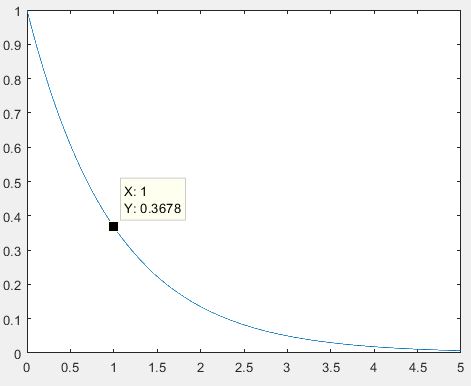
## **1.1 Understanding What a Derivative Is with Visuals**

Let’s take a look at some graphs to help visualize the concept of a derivative. We are familiar with straight lines having slopes as seen in Figure 1.1 below with a slope of -1.



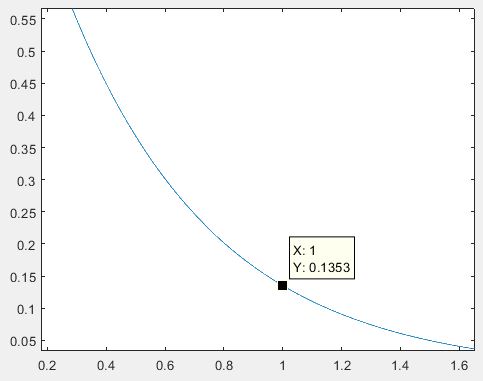
### **Figure 1.1:** Linear Graph with Slope of -1

But what if we had a graph that was not a straight line? Looking at Figure 1.2 below, we have such a graph. Focus on the fact that the graph is curved and does not have a constant slope like the previous graph.



### **Figure 1.2:** Graph with Point at (1, 0.1353)

As stated before, a derivative can be thought of as the change between two infinitely close points. Let’s zoom into the graph to see what it looks like.



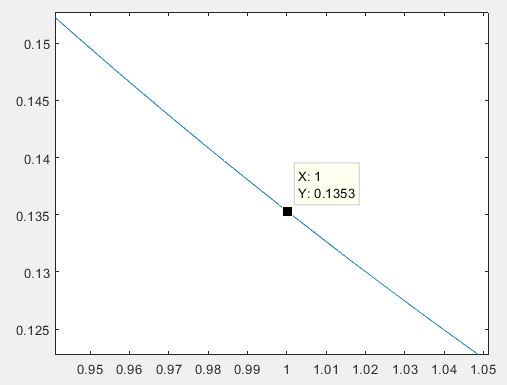
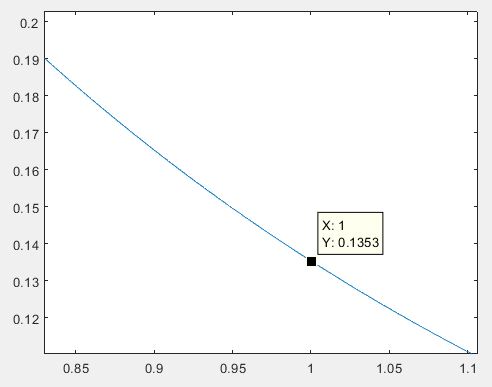
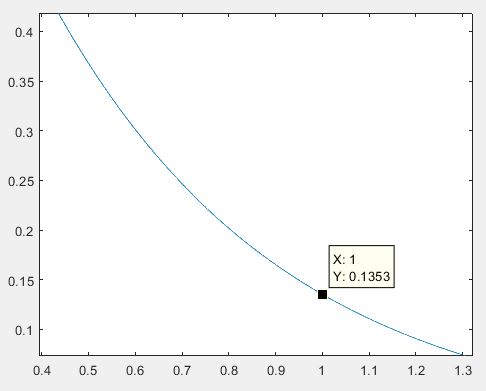
### **Figure 1.3:** A Zoomed-In View of Figure 1.2

Notice from Figure 1.3 that despite being zoomed in, the graph appears to be similar in shape as before. However, if we keep zooming in, we will notice that the graph starts to look more like the straight line in Figure 1.1.

**c**

**b**

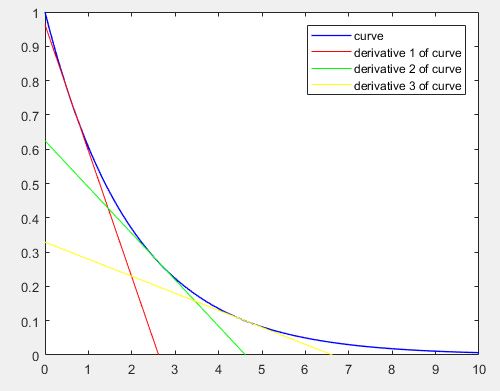
**a**



### **Figure 1.4:** Pictures Showing More Zoomed-In Views of Figure 1.2 (zoom increases from a to c)

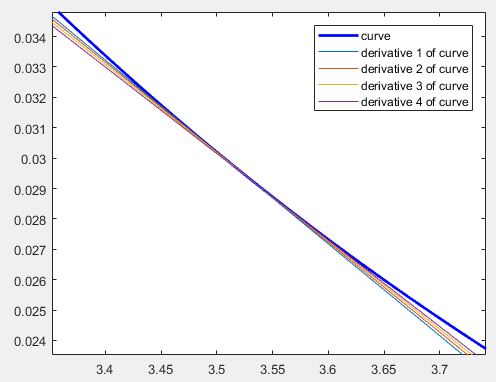
Now, we can start to see that even through a graph may not be straight, if we zoom in enough, the graph becomes essentially a straight line (Figure 1.4c). From this, we can get a slope, but it would be the slope for a very small part of the graph.

If we were to move around the graph and get the slope at different parts, it would not be the same (Figure 1.5). This makes sense because the graph is originally not straight; it is curved. Therefore, it should have a changing slope.



### **Figure 1.5:** Derivatives at Different Parts of Curve

If we tried to zoom in on a straight line, the slope would not change, because the line is not curved. Its slope is constant throughout the line, whereas a curved line will always have a changing slope between different parts of its graph. No matter how close you may zoom in on a curved graph, it will have a different slope for every pair of infinitely close points you take as seen below in Figure 1.6. While the difference between the slopes may be small, perhaps even infinitely small, it is still a difference.



### **Figure 1.6:** Different Slopes for Neighboring Pairs of Infinitely Close Points

As you can see, a derivative is simply the change, or slope, between two really close points. Even the smallest shift in those two points will change the slope of a curve.

# **What is an “integral?”**

## **2.0 A Simplified Definition of an Integral**

An integral is one of the main ideas belonging to Calculus, a branch of mathematics. An integral can be thought of as the space enclosed by something

## **2.1 Understanding What an Integral Is with Visuals**

Genus: Fundamental Idea of Calculus, a branch of mathematics

Species: Integrals

Characteristics: Area, sum of infinitely small width rectangles